Name:

Smart Grids Technologies
Module 4 Lab 2
Stochastic Optimal Power Flow

1 Organization

1.1 Objectives

This lab session covers the basics of stochastic optimal power flow. We assume you have completed the previous labs and are familiar with the different formulations for the loadflow (the BFM and BIM models) as well as the application of these methods to solve a deterministic optimal power flow problem. In this lab you will learn how to implement and solve a stochastic and a robust optimal power flow. The lab aims to show how uncertainty can be dealt with in optimization problems. First, we recall the different methods discussed during the lectures to handle uncertainty in optimization problems. Then the optimal power flow problem considered is presented and the system for which computations will be carried out is presented. You are provided with a python jupyter notebook. The system is given, together with the boundary conditions, constraints on the resources and basic functions to compute the load flow and sensitivity coefficients. Additionally, an example for the robustification of constraints is already given. You will be asked to code the missing pieces, perform some optimal power flow calculations, and interpret the obtained results.

1.2 Evaluation

After completing the lab session, you will be asked to answer quiz questions through Moodle. These questions will serve as evaluation. The lab should be submitted by Sunday 25th of May at 23:55. The Quiz will take place on Monday 26th of May at 9:15.

2 Theory

This section of the lab introduces the optimal power flow problem and the different methods discussed during the lectures to solve it. If you are already familiar with the topics discussed here, feel free to skip this section, and proceed directly to Sec. 3.

2.1 Optimal Power Flow Problems under uncertainty

During the lectures, you have seen different approaches to deal with uncertainty in optimization problems and optimal power flow in particular. In this lab, we will consider both robust formulations, where optimization is performed with the worst case in mind, and stochastic optimization, where the expected outcomes are considered for the objective, while accounting for uncertainty for the satisfaction of constraints. In a general form, a robust optimal power flow problem can be written as:

$$\min_{x} \max_{u} C(x, u) \tag{1}$$

$$\text{s.t.} \quad \text{resource constraints} \tag{2}$$

$$F(u, d) \in \mathcal{X} \quad \forall d \in \mathcal{D} \tag{3}$$

where u represents the set of optimization variables considered in the problem and d represents the stochastic variables. The mapping $F(u, d)$ the state variables which must be within the feasible set $\mathcal{X}$. In the lectures, methods have been described to eliminate the stochastic variables (and thus the quantifiers) and obtain equivalent robust constraints for the optimization variables.

In a general form, a stochastic optimal power flow problem can be written as:

$$\min_{u \in \mathcal{U}} \quad \mathbb{E}[C(u, d)] \tag{4}$$

$$\text{s.t.} \quad \text{resource constraints} \tag{5}$$

$$\mathbb{P}(F(u, d) \in \mathcal{X}) \geqslant 1 - \epsilon \tag{6}$$

Through knowledge of the probability distribution of the stochastic variables, one can transform chance constraints, such as the one describing the stochastic optimal power flow problem in an equivalent robust constraint, which guarantees the constraint is satisfied for all realisations belonging to the set $\mathcal{D}$. If the probability of a stochastic variable d to belong to the set

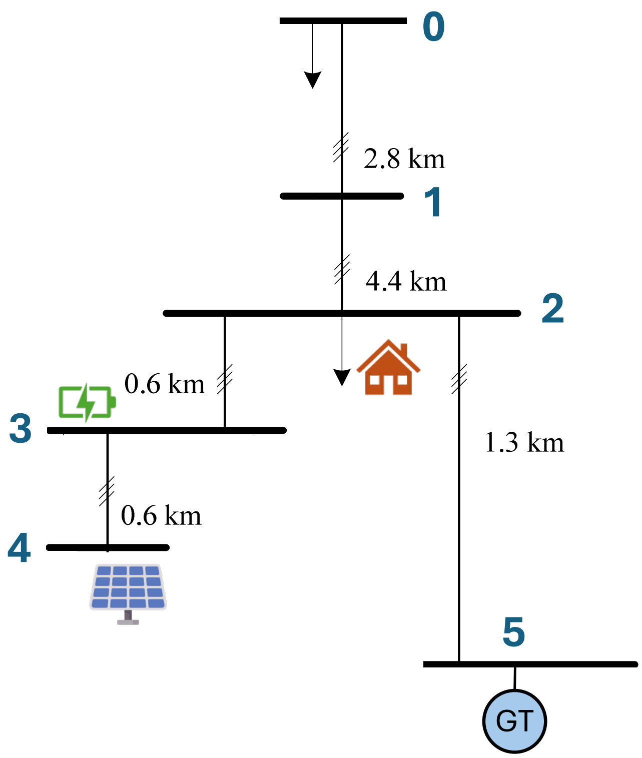


Figure 1: Considered Electrical Network

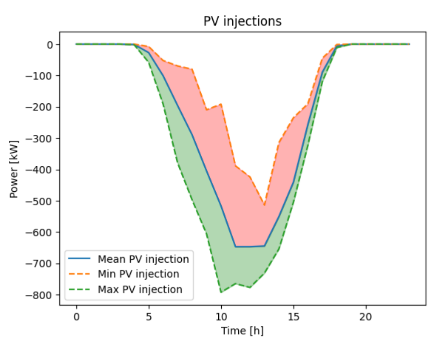
$\mathcal{D}$ is $1-\epsilon$, this is equivalent to satisfying the chance constraint. The obtained constraint can then be transformed to an equivalent robust constraint involving only the optimization variables.

2.2 Considered Optimal Power Flow Problem

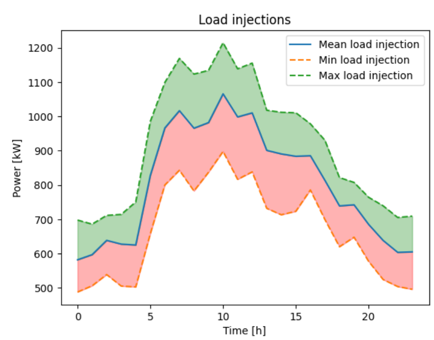
The network considered for the optimal power flow problem in this lab is a subset of the CIGRE MV benchmark that was used during the previous lab. The considered system is visualised in Figure 1. The system consists of six nodes and five lines. Controllable resources are present in the form of a battery and a gas turbine and the network is connected to a main grid in node zero. The battery and gas turbine are modelled in the same way as in the previous lab. This has already been implemented. In this case, we assume the medium voltage network we consider is connected to a weak grid, meaning that the power exchange at the slack node is severely limited to $+/-0.5MW$. The goal of the optimal power flow problem, is thus to minimize the operating cost of the network, while ensuring the power exchanged at the slack node does not exceed these limits and all other grid constraints are satisfied. We assume here that the cost for importing and exporting electricity is the same. This ensures the electricity exchange contribution is a linear function of the slack power. The objective of this optimal power flow problem is the following:

$$\sum_t (c^{el}(t)(P_s^+(t) - P_s^-(t)) + c^{gas}\frac{P_g(t)}{\eta^{GT}})\Delta t \tag{7}$$

The grid model used is a linearised sensitivity coefficient model. The stochastic comsumption represent the consumption of a residential neighbourhood in node two and a large PV installation in node four. These stochastic power consumptions are represented in Figure 2. In this lab, we consider both a stochastic and a robust OPF problem. To avoid coding the constraints twice, we will assume in the stochastic OPF that we want to satisfy the constraints with 100% probability. The uncertainty set for the stochastic variables will then be the same for both the stochastic and robust problem. The only difference between both problems is the objective.



(a) PV Injections



(b) Load Injections

Figure 2: Stochastic Power Consumption

Note that for the net nodal injections, we consider the same convention as during the last lab. The net injected power at a given node is equal to the controllabel injection minus the uncontrollable demand (which is negative for a PV injection). Thus, we have $P = P_n - P_{dn}$ (and equivalently for the reactive power).

We recall the grid model used: Assuming the system was linearized around a certain state $\bar{V}^*$ and sensitivity coefficients for the voltage and current magnitudes at every timestep were obtained as $\boldsymbol{K_{P,V}(t)}, \boldsymbol{K_{Q,V}(t)}, \boldsymbol{K_{P,I}(t)}, \boldsymbol{K_{P,I}(t)}$, we can write the constraints on the voltage and current magnitudes as:

$$V_{min} \leqslant |\bar{V}^*(t)| + \boldsymbol{K_{P,V}(t)} \cdot (P(t) - \bar{P(t)}) + \boldsymbol{K_{Q,V}(t)} \cdot (Q(t) - \bar{Q(t)}) \quad (8)$$

$$V_{max} \geqslant |\bar{V}^*(t)| + \boldsymbol{K_{P,V}(t)} \cdot (P(t) - \bar{P(t)}) + \boldsymbol{K_{Q,V}(t)} \cdot (Q(t) - \bar{Q(t)}) \quad (9)$$

$$I_{min} \leqslant |\bar{I}^*(t)| + \boldsymbol{K_{P,I}(t)} \cdot (P(t) - \bar{P(t)}) + \boldsymbol{K_{Q,I}(t)} \cdot (Q(t) - \bar{Q(t)}) \quad (10)$$

$$I_{max} \geqslant |\bar{I}^*(t)| + \boldsymbol{K_{P,I}(t)} \cdot (P(t) - \bar{P(t)}) + \boldsymbol{K_{Q,I}(t)} \cdot (Q(t) - \bar{Q(t)}) \quad (11)$$

Additionally, we can compute sensitivity coefficients approximating the losses as a function of the nodal power injections. With these coefficients, the power at the slack can be computed as:

$$P_s(t) + \sum_{n\neq 0} (P_n(t) - P_{dn}(t)) = \bar{L_p} + \boldsymbol{K_{P,P_s}(t)} \cdot (P(t) - \bar{P(t)}) + \boldsymbol{K_{Q,P_s}(t)} \cdot (Q(t) - \bar{Q(t)}) \quad (12)$$

$$Q_s(t) + \sum_{n\neq 0} (Q_n(t) - Q_{dn}(t)) = \bar{L_q} + \boldsymbol{K_{P,Q_s}(t)} \cdot (P(t) - \bar{P(t)}) + \boldsymbol{K_{Q,Q_s}(t)} \cdot (Q(t) - \bar{Q(t)}) \quad (13)$$

Additionally, as in this optimal power flow problem we want to enforce bounds on the slack power that hold for all realisations of the stochastic variables, it will be useful to rewrite these equations as:

$$P_s^{min} \leqslant -\sum_{n\neq 0} (P_n(t) - P_{dn}(t)) + \bar{L_p} + \boldsymbol{K_{P,P_s}(t)} \cdot (P(t) - \bar{P(t)}) + \boldsymbol{K_{Q,P_s}(t)} \cdot (Q(t) - \bar{Q(t)}) \leqslant P_s^{max} \quad (14)$$

$$Q_s^{min} \leqslant -\sum_{n\neq 0} (Q_n(t) - Q_{dn}(t)) + \bar{L_q} + \boldsymbol{K_{P,Q_s}(t)} \cdot (P(t) - \bar{P(t)}) + \boldsymbol{K_{Q,Q_s}(t)} \cdot (Q(t) - \bar{Q(t)}) \leqslant Q_s^{max} \quad (15)$$

The quantities P and Q refer to the vector of concatenate injections $P_n - P_{dn}$ for all the nodes in the system (except for the slack node which is left out as the voltage is considered fixed). $\bar{L_p}$ and $\bar{L_q}$ represent the active and reactive losses at the linearisation point.

2.3 Robustification of the constraints.

One of the main tasks of this lab is to obtain equivalent robust constraints, by eliminating the stochastic variables. Here, we show the example of the voltage magnitude constraint (upper limit). In terms of our controllable and uncontrollable power injections, the constraint is:

$$|\bar{V}^*(t)| + \boldsymbol{K_{P,V}} \cdot (P_n - P_{dn} - \bar{P}) + \boldsymbol{K_{Q,V}} \cdot (Q_n - Q_{dn} - \bar{Q}) \leqslant V_{max}, \forall (P_{dn}, Q_{dn}) \in \mathcal{D} \tag{16}$$

This constraint must hold for every node and at every timestep. To ensure any realisation of the stochastic variables satisfies the contraints, each constraint must be considered separately. Thus for node i, we can write:

$$|\bar{V}^{i*}(t)| + \boldsymbol{K^i_{P,V}} \cdot (P_n(t) - P_{dn}(t) - \bar{P}(t)) + \boldsymbol{K^i_{Q,V}} \cdot (Q_n(t) - Q_{dn}(t) - \bar{Q}(t)) \leqslant V_{max}, \forall (P_{dn}(t), Q_{dn}(t)) \in \mathcal{D}^t \tag{17}$$

where $\boldsymbol{K^i_{P,V}}$ represents the sensitivity of node i with respect to the change in power injection at the different nodes. This constraint can be rewritten as

$$\boldsymbol{K^i_{P,V}} \cdot P_n(t) + \boldsymbol{K^i_{Q,V}} \cdot Q_n(t) \leqslant h^i(t) \tag{18}$$

with

$$h^i(t) = \min_{(P_{dn}(t), Q_{dn}(t)) \in \mathcal{D}^t} V_{max} - |\bar{V}^{i*}(t)| + \boldsymbol{K^i_{P,V}} \cdot (P_{dn}(t) + \bar{P}(t)) + \boldsymbol{K^i_{Q,V}} \cdot (Q_{dn}(t) + \bar{Q}(t)) \tag{19}$$

2.4 Formulation of Stochastic and Robust Objective

The objective is given as:

$$\sum_t (c^{el}(t)(P_s^+(t) - P_s^-(t)) + c^{gas} \frac{P_g(t)}{\eta^{GT}}) \Delta t \tag{20}$$

However, the power exchange at the slack node depends on the realisations of the uncontrollable variables. In the stochastic approach, the problem minimizes the expected value of the objective function. This can be done by explicitly replacing the slack power values by their expected value or by writing the objective as is and adding an extra constraint enforcing the variables Ps^+ and Ps^- to be the slack power corresponding to the expected value of the stochastic variables. In a robust problem, we minimize the objective function for the worst cost. Again this can be done by replacing directly the objective function or by writing the constraint(s) that ensure the variables Ps^+ and Ps^- correspond to the worst case realisation of the stochastic power injections.

3 Questions

You are now asked to answer the following questions. For questions requiring only coding, you do not need to copy the code in this report, but you may simply upload the code together with your report in the assignment.

Q1/ Write a function that constructs the equivalent robust voltage constraints. Hint: look at the function for the current constraints and how it is used further in question 5.

[A1]

Q2/ Write a function that constructs the equivalent robust slack power.

[A2]

Q3/ Write a function that constructs the objective function and auxiliary constraints for the stochastic optimal power flow.

[A3]

Q4/ Write a function that constructs the objective function and auxiliary constraints for the robust optimal power flow.

[A4]

Q5/ Solve the stochastic optimization problem. Observe the results, are the constraints satisfied? What is the value of the objective function?

[A5]

Q6/ Solve the robust optimization problem. Observe the results, are the constraints satisfied? What is the value of the objective function?

[A6]

Q7/ Compare the solution of the robust and stochastic optimal power flow problems? Which one has a higher objective? Why? How do the controllable power injections compare? Can you explain this?

[A7]

Add a term $\sum_t c^{el}(0)||P_n(t)||_2$ to the objective function in both cases and solve the optimal power flow problems once more.

Q8/ Compare the solution of the robust and stochastic optimal power flow problems? What has changed? Can you explain why the error for the slack power is lower in the stochastic case? Hint: the slack power is computed for the objective as in the constraints, the value depends on the realisation of the stochastic variables. This means the slack power computed corresponds to the realisation considered in the objective.

[A8]